Regulation During Challenge: A General Model of Learned Performance Under Schedule Constraint

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This article develops a general behavior-regulation model of learned performance related to the equilibrium approach of Timberlake (1980) and Timberlake and Allison (1974). The model is based on four assumptions: (a) Both the instrumental and contingent responses are regulated with respect to their own set points; (b) these set points can be measured in a free baseline when both responses are relatively unconstrained and simultaneously available; (c) a reinforcement schedule can be seen as a constraint function that cross-couples the environmental effects of regulatory systems underlying the instrumental and contingent responses, thereby challenging their set points; and (d) molar behavior change under a schedule represents a compromise between the deviations from set points forced by the constraint function. These assumptions are translated into a set of coupled differential equations describing two regulatory systems related by a schedule. After providing an exact solution for this model, we derive as special cases two current alternative models of learned performance (Allison, 1976; Staddon, 1979). Finally, we demonstrate that the model is consistent in form with data from a variety of simple schedules.

The study of learned performance in animals has roots in at least two traditions: the study of associations, and the study of the adaptive, regulatory functions of behavior. Until recently, research has focused primarily on the nature and the strength of associations formed through experience. Regulatory effects have been largely ignored or masked by the use of highly deprived subjects working during short sessions under stingy schedules. The renewed interest in a regulatory approach can be attributed to several important advantages. In contrast to associative theories, a regulatory approach can identify potential reinforcers in an a priori and noncircular fashion, predict characteristics of asymptotic performance under schedules, and be used to select schedules that produce reinforcement, punishment, or no change in behavior (Timberlake, 1980; Timberlake & Allison, 1974).

Moreover, it can be argued that an accurate regulatory model of learned performance is a necessary prerequisite for evaluating the contribution of associative factors. Certainly in terms of an organism's evolutionary and individual history, learning most often serves within a regulatory framework. The capacity to learn evolved and is typically expressed as an adaptive mechanism to promote survival and reproduction through the more efficient regulation of an organism's internal and external environment. In this view, regulation precedes and underlies learning, setting the context for its nature and expression.

This article translates the equilibrium approach to the study of learned performance (Timberlake, 1980; Timberlake & Allison, 1974) into the concepts of control theory. From the resultant behavior-regulation theory, we develop a set of coupled differential equations that model behavior under schedule constraint. These equations embody the assumption that learned performance results from the simultaneous regulation of behavior under schedule constraint with respect to the behavioral set points of both instrumental and contingent responding. From the model
we are able to derive as special cases the models of Allison (1976) and Staddon (1979). Finally, we show that the coupled-regulation model can account for a variety of data from simple fixed-ratio, fixed-interval, and variable-interval schedules.

The Regulatory Approach

Basic Elements

The application of a regulatory analysis to behavior began early in this century with work on guided orientation in insects (Crozier & Hoagland, 1934; Loeb, 1918) and physiological homeostasis in mammals (Cannon, 1934). In the late 1940s and 1950s a general control theory was developed in a form that clarified the nature of regulation and the concept of feedback. Beginning in the late 1960s and 1970s researchers such as McFarland (1971) and Powers (1978) applied regulatory concepts to the prediction of choice and sequencing of behaviors. Most recently, researchers have used control theory in describing and modeling the time course of eating and drinking (Booth, 1978; Toates, 1980) and behavior under reinforcement schedules (Baum, 1981; Pring-Mill, 1979; Staddon, 1980).

The basic elements of a regulatory approach are (a) One or more stable set points or reference inputs that the system is organized around and tends to move toward as that system is perturbed, (b) a constraint function that represents challenges that drive the system away from its set points, and (c) a control function that describes exactly how the system responds to the challenge imposed by the constraint function. The regulatory approach is extremely general and can be applied to both physical and biological systems under many different forms of challenge. In this article we will focus on its application to asymptotic learned performance under schedule challenge.

Differential Equation Models

Recent developments in the modeling of behavior have leaned heavily on the use of differential equations, not only in realizations of control theory (e.g., McFarland, 1971; Toates, 1980), but in other theoretical schemes as well (Atkinson & Birch, 1970; McDowell & Kessel, 1979; Myerson & Miezhen, 1980; Rachlin, Kagal, & Battalio, 1980). This trend reflects several advantages that accrue to the modeler when he or she is framing theoretical assumptions in terms of a differential equation. First, a rate-change description often captures intuitions about the mechanisms of behavior more readily than an attempt to describe performance directly. It is easier to mathematically represent processes in terms of discrete “snapshots” than by continuous specification through time. Second, the differential equation approach is powerful in that simple verbal assumptions can often be mapped to and from nonintuitive, nonrational functional forms. For example, assuming a proportional relation in a differential equation results in an exponential function.

Finally, the functional form resulting from a differential equation model is a unique product of the assumptions that define it. Functional equations developed primarily on the basis of intuition or curve fitting are at best consistent with the modeler’s verbal assumptions rather than being forced by them (e.g., Allison, 1976; Herrnstein, 1970; Hull, 1943). A potential problem with such a “consistency” derivation is that the functional form described by the model is very unlikely to represent a unique mapping of underlying assumptions. As a result, fitting tests of a particular model are inconclusive because they cannot distinguish among the alternative sets of assumptions. For example, the form of the matching law (Herrnstein, 1961) has been derived from several different and apparently incompatible sets of assumptions (Baum, 1981; Herrnstein, 1970, 1979; Herrnstein & Vaughn, 1980; Killeen, 1979, 1982a; Myerson & Miezhen, 1980; Staddon, 1977; Staddon & Mother, 1978). Currently, none of these derivations has been shown to be related nor has one been shown to be a unique mapping of any set of basic assumptions underlying matching.

In sum, the use of differential equations allows the tracing of important steps from basic assumptions to functional forms. The fact that the researcher is forced to map assumptions directly to differential equations may provide and suggest critical tests of the model. Differential equations have been used
increasingly to model behavior, although the specific differential equation is often obscured by the absence of a deductive derivation, the particular method of solution (e.g., Laplace transforms—McDowell & Kessel, 1979), or a focus on stationary analysis (e.g., optimization theory—Rachlin, Kagel, & Battalio, 1980). The remainder of this article shows how a general set of differential equations can be developed from a regulatory approach and applied to the prediction of asymptotic learned performance under a contingency.

A Behavior Regulation Approach to Learned Performance

Our model is a more precise statement and generalization of the molar-equilibrium approach outlined by Timberlake (1980), which is, in turn, a more precise and general statement of the response-deprivation approach of Timberlake and Allison (1974). Both of these approaches find their roots in earlier work by Premack (1959, 1965). According to the equilibrium approach, changes in instrumental responding occur when a schedule imposed by the experimenter produces a condition of disequilibrium with respect to baseline levels of responding. The final level of instrumental responding is determined by a balance between the deviations from baseline levels of instrumental and contingent responding.

Translated more fully into the language of regulation, the equilibrium approach entails the following set of assumptions: (a) Both the instrumental and contingent response are regulated with respect to their own behavioral set points; that is, in the absence of a schedule, the instrumental and contingent responses are controlled by separate regulatory systems. (b) Behavioral set points for the instrumental and contingent responses can be measured in a free baseline when both responses are relatively unconstrained and simultaneously available. (c) A schedule can be viewed as a constraint function that couples the environmental effects of the outputs of the regulatory systems underlying instrumental and contingent responding. A schedule effective in producing learned performance couples the systems so that the organisms cannot achieve both set points at once. (d) The change in each response under the schedule is determined by simultaneous regulation of both systems with respect to their set points.

In the next three sections we develop more completely the control-theory view of equilibrium theory by considering the nature of behavioral set points, constraint functions, and control functions as revealed in these assumptions. The following section then translates the resultant behavior-regulation theory into a set of differential equations suitable for modeling learned performance.

Set Points

In psychology, regulatory theory has focused on physiological set points that appear to be involved in such homeostatic mechanisms as salt balance, metabolism, and blood volume (Cannon, 1934). In this article we extend the concept of set points to refer directly to behavior (Timberlake, 1980). Instrumental and contingent responding are assumed to be regulated with respect to their total unconstrained responding or, more precisely, with respect to the instigation producing this responding. Undoubtedly, this instigation reflects the existence of more complex underlying physiological processes that may or may not be regulatory (cf. Bolles, 1980), but for our purposes it is necessary only that the instigating value of any underlying processes be reliable.

Most behavior-regulation theories share the assumption that set points of behavior can be measured by total responding in a paired or multiple baseline in which the instrumental and contingent responses are relatively unconstrained and simultaneously available. These baseline measures are presumed to indicate the degree of instigation or arousal (cf. Killeen, Hanson, & Osborne, 1978) associated with these behaviors during minimal constraint. As constraints are added, responding becomes less related to the fundamental level of instigation for a particular response and more determined by its linkage with the other response and by various "expression" characteristics that modify the degree of change under a particular challenge (e.g., elasticity—Staddon, 1977; resistance—
Nevin, Mandell, & Atak, 1983). Asymptotic responding is determined both by fundamental levels of instigation and by the expression characteristics of a response. However, although expression characteristics can be established under conditions of constraint, the fundamental levels of instigation are more properly measured in the absence of constraint.

It might be argued that baseline measures are not really set points because they may vary with the instigating qualities of the circumstances. The important point here is that behavioral set points are useful only if the fundamental levels of instigation they reflect remain present. Thus, experimental procedures are designed to hold instigation constant across sessions and conditions by fixing variables such as session length, physical environment, time of day, and the subject's physiological state. The only change in conditions typically allowed is the imposition of a schedule constraint following baseline assessment. Note also that altering the eliciting conditions does not invalidate the regulatory approach; it simply requires remeasuring baselines or, somewhat less happily, fitting the baselines from the contingency data. Ultimately, the baseline method of defining set points will give way to a behavioral theory of instigation, but, for the present, the paired baseline serves as a reasonable approximation of instigation levels underlying responding.

**Constraint (Schedule-Feedback) Functions**

In regulatory theories a constraint function determines the relation between the output of a system and its resultant input. The constraint function is not an intrinsic part of the regulatory system but depends upon the environment in which the system is operating. In learned performance there are at least two types of constraint function, one based on the environmental circumstances that control and interact with a particular response, for example, the location, weight, and resistance of a bar (essentially, the "measurement system"), and one connecting the measured performance of two responses. The first type of constraint function is typically invariant across the experiment and is incorporated in the baseline assessment of set points. The second type of constraint function refers to the relation imposed by the experimenter between the instrumental and contingent responses. It is this constraint that challenges the set points and determines learned performance. Many recent theorists have termed this relation a *schedule-feedback function*.

The schedule-feedback function is a rule relating performance of the instrumental response to the amount of reward. For example, the schedule-feedback function for a fixed-ratio-five schedule is quite simple: Pressing the bar five times produces access to one pellet of food. The overwhelming majority of current theories of learned performance depend on the specification of schedule-feedback functions to generate predictions of absolute levels of performance under schedules. For example, optimality theories have used the schedule-feedback function to constrain the range of molar output so that responding must fall somewhere on this function. The optimality equation then determines where the responding will actually fall.

Unfortunately, there are problems with this dependence on the schedule-feedback function. First, implicit in such dependence is an assumption that unique schedule characteristics are critical determinants of molar performance. It is clear that momentary response patterns may be determined by specific schedule characteristics; however, it is possible to consider molar output as independent of these details. For example, an alternative assumption is that organisms adopt strategies based on general constraints that arise in any schedule. Thus, over a session a stable set of behavioral strategies could emerge that are relatively insensitive to the particular schedule characteristics. The schedule-feedback function adopted by Staddon and Matheral (1978) for the variable-interval schedule illustrates how aspects of responding can be independent of specific schedule characteristics. According to their schedule-feedback function, the organism was assumed to produce random interresponse times.

The second problem with dependence on the schedule-feedback function is the disagreement about its precise form. Consider,
for example, the case of the variable-interval schedule. The schedule constraint in this case is quite weak: Both the number of responses and their exact location in time are left uncontrolled; a response need only occur after a variable interval to produce reward. As a result, the response output can be quite complicated to specify, depending upon the characteristics of interresponse times (cf. Nevin & Baum, 1980). In fact, the exact characterization of responding has been a source of considerable disagreement in specifying the precise form of the schedule-feedback function for variable-interval schedules (Heyman, 1979; Heyman & Luce, 1979; Rachlin, 1978; Staddon & Motheral, 1978, 1979).

The use of approximations to obtained rate of reinforcement potentially sidesteps this problem. Even so, it is hard to know which approximation to use. Myerson and Miezen (1980) assumed that obtained rate was reasonably approximated by programmed rate. Staddon and Motheral (1978) assumed that obtained rate was determined by a Poisson process that tracked programmed rate. Nevin and Baum (1980) offer a closer approximation to obtained rate based on a complex burst–pause random process. Given these alternatives, it is possible that predictions from the same model will differ as a function of the approximation. There is a clear danger in basing a model on such a variable cornerstone. If, on the other hand, predictions from a model do not vary under such a range of approximations, then the precise designation of the schedule-feedback function may be unnecessary.

In the present approach we have chosen the alternative of eliminating the specific details of the schedule-feedback function. We have preserved only the effect of the schedule as a molar constraint that couples or relates the instrumental and contingent responses. Therefore, we specify the schedule-feedback function as an unknown, unique transformation of the schedule imposed by the experimenter. Given a particular allocation of time to one behavior, \( t_2 \), the level of the other behavior, \( t_1 \), is given by

\[
  t_1 = h(t_2; p),
\]

where \( h \) is the constraint function describing the actual relation of the two behaviors under the schedule and \( p \) is the rate of exchange between the two behaviors specified by the experimenter.

This treatment of the schedule-feedback function has three immediate consequences. First, because the exact nature of the constraint function need not be specified, we may develop a general model of learned performance that should suffice for any schedule. Second, such a general model can provide a framework for exploring proposed schedule-feedback functions. Third, responding cannot be predicted as a function of specific characteristics of the schedule, such as programmed rate of reinforcement or ratio requirement.

**Control Functions**

In a regulatory system a control function determines the output by relating the input to a reference input or set point. Differences between the set point and input produce a signed error signal that determines the direction and amount of change in output. The control function of a system states the precise relation between the error signal and changes in output.

In the case of learned performance there are two regulatory systems, one each for the instrumental and contingent responses; therefore, there are also two control functions. Because of the experimenter-imposed contingency these control functions are not independent. By definition a contingency references the output level of one response to another. This kind of constraint means that the regulatory actions of the response systems are connected or coupled.

Specifically, the control functions associated with each response system cross-couple the error signals for the two behaviors such that the error signal for one response is related to the output of the other. Thus, the output of the instrumental response is coupled to the error signal for the contingent response so that moving the contingent response closer to its set point requires an increase in the instrumental response. Similarly, the output of the contingent response is altered by the error signal for the instrumental response so that moving the instrumental response closer to its set point requires a de-
crease in the contingent response. Stable behavior in such a cross-coupled system depends on compromise between deviations from the two set points involved. A set of differential equations more precisely formulates the coupling in the next section.

The Coupled-Regulation Assumption

General Coupled-Regulatory Equations

We present a set of two differential equations to capture in a general way the assumptions of the equilibrium approach. The first equation describes the change in one of the responses linked by a schedule, and the second describes the accompanying change in the other response. Although we typically call one of the responses instrumental and the other contingent, this designation is a decision made by the experimenter, usually on the basis of which response is relatively constrained. A second important point is that because we chose to represent the constraint function as an unknown function, we cannot specify changes in responding on a moment by moment basis. Rather the equations specify behavior change over the session time, T. However, our general approach is not limited in principle to this molar level of analysis because more specific assumptions about the constraint function may be incorporated into these equations.

Given two activities with time allocation \( t_1 \) and \( t_2 \), with respective set points \( s_1^0 \) and \( s_2^0 \), related by a contingency schedule \( h \), in a fixed time sample \( T \), the set of equations is as follows:

\[
\frac{dt_1}{dT} = f[s_1^0 - h(t_1; p), t_1, s_1^0]
\]

and

\[
\frac{dt_2}{dT} = f[s_2^0 - h^{-1}(t_2; p), t_2, s_2^0].
\]

These are the most general regulatory differential equations that represent changes in behavior under schedule challenge. The first equation says that the change in the time allocated to the first behavior \( (t_1) \) over the session time \( (T) \) is a function of (a) the error signal for the second behavior, that is, the difference between the set point for the second behavior \( (s_2^0) \) and its performance under the schedule \( [h(t_1; p)] \), (b) the set point for the first behavior \( (s_1^0) \), and (c) the level of output of the first behavior \( (t_1) \). The second equation specifies symmetric relationships for the second activity, that is, the change in the second behavior is a function of the error signal for the first behavior, and the set point and level of output of the second behavior. Note that the inverse of the constraint function, \( h^{-1} \), is used to specify the level of output of the second behavior given the first behavior.

The equations are said to be coupled because the changes in one activity are referenced to the deviation of the other activity from its set point. This deviation is transformed by the control function, \( f \), which expresses the relation between the error signal of one activity and the output of the other. If we assume that the function \( f \) preserves signed differences and emphasizes the coupled terms of the equations, it can be seen that this general system subsumes the predictive aspects of molar-equilibrium theory with respect to changes in baseline responding under a schedule (Timberlake, 1980).

If the constraint function, \( h \), allows the subject to achieve only one set point while it constrains the other response, then the derivative of the behavior linked to the constrained response will be positive, producing an increase in its level of responding over the session (reinforcement). On the other hand, if the constraint forces responding over the set point for one behavior, then the derivative of the other behavior will be negative, producing a decrease in its level of responding over the session (punishment). Finally, if the constraint function allows both behaviors to achieve their set points at once, both derivatives will be zero and responding should remain at set-point levels for both behaviors (no change).

Special Coupled-Regulation Equations

To develop a more specific set of equations, we must designate the form of the control function, \( f \). The development of this function requires consideration of several important issues. The first is the representation of the constraint function, \( h \). Because it represents the level of performance of one behavior at
any moment with respect to the other behavior, we can replace it with the actual level of performance of the behavior in question. Thus, the deviation of the first behavior from its set point (the error signal) can be written as \( s^0_1 - t_1 \). The second issue is the role of the present level of a behavior in scaling the effects of the coupled-error signal on responding. A reasonable assumption is that the effects of the coupled-error signal on changes in responding are directly proportional to its output level. Thus, the effects of the error signal for the second behavior on the change in the first behavior can be written as \( (s^0_2 - t_2)l_2 \). In other words, the more time an organism invests in a particular behavior, the greater the change in that behavior for a particular coupled-error signal.

The third issue is the role of the set point of a behavior in modifying its own change in responding. It is important that the change in a behavior be related to its own set point because a fundamental assumption of the equilibrium approach is that each behavior is controlled by a separate regulatory system. Although the coupling in the regulation equations cited clearly adds other determinants of behavior change, it would not be expected to remove entirely the importance of the uncoupled response determinants. We considered a variety of ways in which a response's own set point could modify its responding over time. Many of the forms that we considered served at least partly as a resistance to increases in responding that varied directly with the level of output. We eventually chose the ratio of the expression of a response to its set point \( \left( \frac{t_1}{s^0_1a_1} \right) \), in part because of its relative simplicity and tractability. According to this expression the resistance to increase of a response is inversely related to its set point, and is directly related to its current response level.

As we show in the equations ahead, this form of including the set point of a behavior means that its contribution to its own change is asymmetric with respect to the direction of movement. Other things being equal, behaviors with high set points will increase more readily and decrease less readily under a schedule and tend to be more stable over time in a baseline condition. In contrast, responses with low set points show larger resistance to increase and less resistance to decrease under a schedule, and tend to drift exponentially toward lower levels of responding in a baseline condition. A final important issue is how to reference the behavior change predicted by these equations. To clarify the ability of the general set of equations (Equation 2) to subsume the predictions of equilibrium theory, we referenced the changes in behavior over the session time to the baseline values. In the current equations we reference the changes at any time \( t \) to the zero levels of responding at the beginning of the session interval.

Combining the general coupled equations above with our specification of the control functions relating the inputs and outputs of the two behaviors produces the set of equations that follows. Given two behaviors with times of expression equal to \( t_1 \) and \( t_2 \), setpoints of \( s^0_1 \) and \( s^0_2 \), and a constraint function cross-coupling the outputs such that the second behavior is relatively deprived, the change in each behavior at any moment in the session time, \( t \), can be written as

\[
\frac{dt_1}{dt} = b(s^0_2 - t_2)l_1 - \frac{t_1}{s^0_1a_1}
\]

and

\[
\frac{dt_2}{dt} = -\frac{1}{b} \left( s^0_1 - t_1 \right)l_2 - \frac{t_2}{s^0_2a_2},
\]

where \( a_1 \) and \( a_2 \) represent each behavior's resistance, and the amplification factor or linkage is represented by \( b \). These equations are not as general as the first set in that it is necessary to enter the relatively unconstrained (instrumental) response in the first equation and the relatively constrained (contingent) response in the second equation. The negative sign emphasizes that as the performance for one behavior increases the other should decrease (cf. Myerson & Miezen, 1980). It also causes both the instrumental and contingent responses to increase through time even though error signals of these responses have opposite signs.

The parameters of this equation have reasonable interpretations as scaling factors. For example, the parameter \( b \) modifies the contribution of the error signal from the coupled behavior and appears likely to be related to the strength of the motivational linkage be-
between the two responses. In other words, $b$ is at least partly an index of the motivated association between the responses and, thus, should vary with characteristics of the schedule. Schedules that specify close relations between instrumental and contingent behaviors would be more likely to have high values of $b$. In the present form of the model, we made the simplifying assumption that this linkage is reciprocal for the two responses, that is, if the effect of one coupled-error term is modified by $b$, the effect of the other is modified by $1/b$. Whether this simplification is broadly useful remains to be seen.

The parameters $a_1$ and $a_2$ scale the resistance of the two responses to increase. These values reflect some of what we previously called the expression characteristics of a response. Those aspects of responding have received recent attention in concepts such as elasticity (Rachlin, Battalio, Kagel, & Green, 1981; Staddon, 1977) and behavioral inertia or momentum (Nevin, 1974; 1979; Nevin et al., 1983). In the present model, expression characteristics should modify changes in behavior under schedule constraint as well as stability of baselines when the responses are uncoupled.

Under uncoupled conditions, according to these equations, the expression of each response will drift exponentially downward under fixedinstigation and time toward an output level proportional to its set point. If an increase in instigation occurs, a response should increase therefore in a fashion inversely proportional to its set point and with a degree of hysteresis (relative to its return) that is directly proportional to its set point. Under coupled conditions, this differential system represents the tension between the set points of the two responses and the requirements of the schedule-constraint function. The solution to such a system results in a functional relation between the two responses. The plot of this function, the molar output of one response against the other, is called the phase plane of the system. Given any initial conditions, all solutions to the differential equations can be shown in the phase plane. In the present case, therefore, the phase plane represents the predictive space of the theory, and the shape and form of the data in the phase plane provide an important initial test of the model.

Solutions

The intent of this section is to derive and represent several solutions to this system of differential equations. The basic form of a solution is a functional relation between the two behaviors. We first outline three approaches to producing such a solution, and then represent the solutions in a phase plane. The first solution is an exact form; that is, it represents a complete solution to these equations. Unfortunately, the form is not a simple analytic function. The second and third solutions are based on standard approximations to an exact solution. They reveal critical properties of a system of differential equations and are especially useful when no exact form can be found. These other solutions are particularly interesting because they represent current models of Allison (1976) and Staddon (1979).

Exact Form

Because these equations include the non-linear term $t_1t_2$, they would ordinarily prove quite difficult to solve—requiring some sort of approximation technique (e.g., Runge-Kutta). Fortunately, this system is similar in form to the well-known Lotka-Volterra system (Lotka, 1956), which has a relatively simple solution and a well-understood behavior. The Lotka-Volterra system has been used with some success in modeling intraspecies competition and predator-prey relations. The solution to the system is relatively straightforward and is repeated here.

Eliminating the time variable, $t$, and considering the derivative of $t_1$ with respect to $t_2$ yields

$$\frac{dt_1}{dt_2} = \frac{b(s_2^0 - t_2)y_1 - \frac{t_1}{s_2^0a_1}}{-\frac{1}{b}(s_1^0 - t_1)y_2 - \frac{t_2}{s_2^0a_2}} \quad (4)$$

This equation is part of the class of separable differential equations (Ross, 1974) and therefore each variable and derivative may be isolated by multiplication or division. After iso-
lating each variable we may integrate both sides of the equation:

\[-\frac{s_0}{b} \log t_1 + \frac{t_1}{b} - \frac{1}{s_0^2 a_2} \log t_1 + C1 = \]

\[bs_0^2 \log t_2 - bt_2 - \frac{1}{s_0^2 a_2} \log t_2 + C2, \quad (5)\]

where \(C1\) and \(C2\) are constants of integration. Factoring this equation, collecting terms on one side, and letting \(C1 - C2 = c\) produces the following solution:

\[\left[\left(\frac{bs_0^2}{s_0^2 a_2} - \frac{1}{s_0^2 a_2}\right) \log t_2 - bt_2\right]

+ \left[\left(\frac{s_0}{b} + \frac{1}{s_0^2 a_2}\right) \log t_1 - \frac{t_1}{b}\right] = c. \quad (6)\]

Exponentiating (letting \(e^c = k\)) reveals the complex structure of this function:

\[\frac{t_2 \left((bs_0^2 - \frac{1}{s_0^2 a_2}) \log t_2 - bt_2\right)}{t_1 \left(b \left(\frac{s_0}{b} + \frac{1}{s_0^2 a_2}\right) \log t_1 - \frac{t_1}{b}\right)} = k. \quad (7)\]

The exponential terms on each side of the equation do not allow the isolation of either variable. In fact, these so-called "transcendental equations" (Southworth & Delleuwe, 1965) cannot be reduced to a simple analytic form [neither \(y = f(x)\) nor \(x = f(y)\)]. This property makes regression analysis difficult. Moreover, the simple behavior of the function in the phase plane has to be determined through some approximation technique.

Two other techniques reveal properties of the differential system and phase plane without an analytic solution. The first identifies the stationary behavior of the system (optimal solution) and allows the location of "critical points" around which the phase plane is organized. In nonlinear systems this stationary analysis provides important qualitative information about the system prior to using a numerical technique to reveal its detailed behavior (Burghes & Borrie, 1981).

A second technique called linearization may be used to gain an intermediate level of information between a stationary analysis and an analytic solution. By linearizing the differential equations, the intrinsic non-linearity of the system may be explicitly ignored, leaving a linear system whose solution provides conservative information about the system. That is, the linearization produces a subset of the behavior of the nonlinear system by providing an approximation of the exact functional form of the solution to the differential equation (Equation 7).

In addition to providing more information about the system of equations, both of these techniques lead to special cases of the differential system that are related to other recently proposed models of learned performance. The stationary solution of the coupled-regulation model can be used to derive Allison's (1976) linear conservation model, whereas the linearization of the coupled-regulation model can be used to derive Staddon's (1979) minimum-distance model.

**Stationary Analysis: Linear Models**

As already mentioned, the phase plane for a set of differential equations is a plot of the two variables independent of time. The critical points organize the phase plane and determine the direction and movement of the system from various starting points. Critical points are defined as those values of the variables that cause the derivatives simultaneously to vanish.

For the present model here are four such cases. The first is a trivial case—when both \(t_1\) and \(t_2\) simultaneously vanish. Thus, the origin is a critical point, and initial or starting values of the system close to the origin tend to be drawn into the origin (see Figure 1). The next two cases do not have a simultaneous solution and therefore do not define critical points:

\[t_1 = 0 \quad \text{and} \quad b_1(s_0 - t_1) - \frac{1}{s_0^2 a_2} = 0\]

or

\[t_2 = 0 \quad \text{and} \quad b_2(s_0^2 - t_2) - \frac{1}{s_0^2 a_1} = 0. \quad (8)\]

The last case defines a locus of critical points in the phase plane and occurs when

\[b_1(s_0^2 - t_2) - \frac{1}{s_0^2 a_2} = 0\]

and

\[-b_2(s_0^2 - t_1) - \frac{1}{s_0^2 a_1} = 0. \quad (9)\]
Solving these equations simultaneously results in a decreasing linear function between $t_1$ and $t_2$ with slope equal to the ratio of the instigation parameters and intercept equal to a complex function of the baselines, instigation, and resistance parameters:

$$t_1 = \frac{1}{b_2} \left[ b_1 s_2^0 + b_2 s_2^0 + \frac{1}{s_2^0 a_2} - \frac{1}{s_1^0 a_1} \right] - \frac{b_1}{b_2} t_2$$

(10)

This linear function represents an optimal compromise relative to the instigation and resistance of each behavior as it changes along some constraint function.

As an outcome of the coupled-regulation model, the linear function would be likely to arise when (a) the constraint function is relatively simple as, for example, in the case of proportionality, (b) the range of each behavior is restricted so that resistance of either response has little effect on output, and (c) the behaviors have relatively low resistance to change. If enough of these conditions are satisfied, the behavior of an organism should be predictable from a linear model.

Allison (1976) proposed a linear model of molar performance based on the notion that some dimension of the behavior is conserved during baseline and contingency sessions. The optimal solution of the coupled regulation is identical to Allison’s conservation model when

$$k = \frac{b_1}{b_2}, \quad \frac{1}{s_1^0 a_1} = \frac{1}{s_2^0 a_2},$$

(11)

and in our notation appears as

$$t_1 = s_2^0 + k s_2^0 - k t_2.$$  

(12)

Thus Allison’s model—assuming we provide appropriate interpretations for the parameters—can be shown to be an optimality model based on the present regulation approach.

Linearization: Elliptical Models

The behavior of linear differential systems is well understood, and these equations are usually straightforward to solve. Thus, a nonlinear differential system may be simplified by focusing on its linear terms. This approach provides both a simple solution and an approximation of the behavior of the complete nonlinear system.

The linearization can be accomplished by considering the Taylor expansion of the nonlinear solution (Equation 6) around two arbitrary positive constants, $k_1$ and $k_2$:

$$\left( b_1 s_2^0 + \frac{1}{s_2^0 a_2} \right) \left( \log k_2 + \frac{(t_2 - k_2)^2}{3 k_2^2} + \frac{(t_2 - k_2)^3}{3 k_2^3} + \cdots \right) - b_1 t_2$$

$$+ \left( b_2 s_2^0 - \frac{1}{s_1^0 a_1} \right) \left( \log k_1 + \frac{(t_1 - k_1)}{k_1} \right)$$

$$- \frac{(t_1 - k_1)^2}{2 k_1^2} + \frac{(t_1 - k_1)^3}{3 k_1^3} + \cdots$$

$$- b_2 t_1 = c.$$  

(13)

As $t_1$ and $t_2$ are in the neighborhood of the origin, we may ignore the higher order terms. Letting

$$k_1 = \frac{1}{b_2} \left( b_2 s_2^0 + \frac{1}{s_1^0 a_1} \right)$$

and

$$k_2 = \frac{1}{b_1} \left( b_1 s_1^0 - \frac{1}{s_2^0 a_2} \right),$$

(14)

the expansion reduces to

$$\left[ t_2 - \frac{1}{b_2} \left( b_2 s_2^0 - \frac{1}{s_1^0 a_1} \right) \right]^2$$

$$+ \left[ t_1 - \frac{1}{b_1} \left( b_1 s_1^0 + \frac{1}{s_2^0 a_2} \right) \right]^2$$

$$+ \left[ \frac{t_1 - 1}{b_1} \left( b_1 s_1^0 + \frac{1}{s_2^0 a_2} \right) \right]^2$$

$$+ \left[ \frac{t_2}{b_2} \left( b_2 s_2^0 - \frac{1}{s_1^0 a_1} \right) \right]^2$$

$$= \text{constant}.$$  

(15)

This is an ellipse with center at $k_1$ and $k_2$, major axis at $4k_1$, and minor axis at $4k_2$.

---

1 We have chosen to write two instigation parameters instead of one. All of the following results would be identical except that the instigation parameter would be squared in some cases.
Thus, the center interacts with the height and width of the ellipse. As the center of the ellipse moves toward the origin, the ellipse grows. Both the height and width of the ellipse increase.

Staddon (1979) proposed a similar elliptical model of learned performance based on the premise that an organism under schedule constraint attempts to minimize the Euclidean distance from the schedule performance to the baseline level. To derive his model Staddon assumed that the schedule constraint had to be introduced as a unique function (we consider only his ratio schedule case) and that the background behavior \(t_3\) varied inversely with the two measured behaviors under a contingency. From these assumptions Staddon derived the standard second-degree equation relating response outputs \(t_1\) and \(t_2\):

\[
\frac{1}{k_1} t_1^2 (a^2 + b^2) + \frac{1}{k_2} (b^2 + c^2) - t_1 (b^2 s_1^0 + b^2 s_2^0 + a^2 s_3^0) - t_2 (b^2 s_1^0 + b^2 s_2^0 + c^2 s_3^0) + 2b^2 t_1 t_2 = 0,
\]

where \(a\), \(b\), and \(c\) represent "costs" of engaging in each activity \((t_1, t_3, \text{and} t_2)\) respectively.

The linearization of our model was shown above to be an elliptical form. At first glance it is not clear that Staddon’s (1979) model is generally elliptical because there are other conics that this second degree equation can define. However, it can be shown generally that Staddon’s model is elliptical and therefore a special case of the coupled regulation model.

First, consider the special case, \(b = 0\), where background behaviors are not considered or where they have insignificant cost for the organism. Staddon’s model immediately reduces to a simple elliptical form:

\[
\frac{(t_1 - \frac{1}{2} s_1^0)^2}{a^2} + \frac{(t_2 - \frac{1}{2} s_2^0)^2}{c^2} = \text{constant.}
\]

In this case, the center of Staddon’s ellipse, the point that defines the maximum, is predicted to be exactly one half of the baseline values, a prediction that differs from the one based on the linearization of our model and that seems arbitrary.

Now consider the general case of \(b \neq 0\). It can be shown that if

\[
2b^2 - 4(a^2 + b^2)(b^2 + c^2) < 0,
\]

then Staddon’s model is always an ellipse as opposed to other second-degree forms like parabolas, hyperbolas or degenerate conics. This condition is always satisfied unless negative costs are allowed.

Placing the \(b\) parameter back into the equation allows the ellipse to rotate in the \(xy\) plane. The \(x\) and \(y\) axes can be rotated so that the \(xy\) term in Staddon’s second-order form disappears, revealing a standard elliptical form:

\[
\frac{(t_1 - k_2)^2}{k_1} + \frac{(t_2 - k_4)^2}{k_3} = \frac{1}{4} \left(\frac{k_2}{k_1}\right)^2 + \frac{1}{4} \left(\frac{k_4}{k_3}\right)^2,
\]

where (standard transformation formulas)

\[
k_2 = \frac{(b^2 t_1 + b^2 s_1^0 + a^2 s_3^0) \cos r + (b^2 s_1^0 + b^2 s_2^0 + c^2 s_3^0) \sin r}{k_1 (a^2 + b^2) \cos^2 r + 2b^2 \sin r \cos r + (b^2 + c^2) \sin^2 r},
\]

\[
k_4 = \frac{(b^2 s_1^0 + b^2 + c^2 s_2^0) \cos r - (b^2 s_1^0 + a^2 s_3^0) \sin r}{k_3 2(a^2 + b^2) \sin r 2b^2 \sin r \cos r + (b^2 + c^2) \cos^2 r},
\]

and

\[
r = \frac{1}{2} \arctan \left(\frac{2b^2}{a^2 - c^2}\right),
\]

if \(a^2 = c^2\) then \(r = \frac{\pi}{4}\). (20)

Thus, the center of Staddon’s ellipse is a complex function of the cost parameters and baseline values. Further rotation of the ellipse is defined by \(r\), which from this last equation can be seen to be a function of all three cost parameters. Also from this form it is obvious that the center of the ellipse interacts with its height and width. As the center moves to-
wards the origin, the ellipse shrinks, that is, both the height and width decrease. This prediction is opposite to the one derived from the linearization of the coupled-regulation model (Equation 15). The critical difference notwithstanding, the form of Staddon’s minimum-distance model is a special case of the coupled-regulation model. For a single data set, Staddon’s model and the linearization model are indistinguishable.

Phase Plane Representation

Figure 1 illustrates the results of the previous three analyses in a phase plane for two behaviors, $t_1$ and $t_2$. Trajectories, indicated by arrows, tend to move toward the decreasing linear function in the middle of Figure 1 (Allison’s model). Near the origin (defined as $k_1$, $k_2$) the system is almost elliptical (Staddon’s model). As we move away from the origin, the system flattens out in some places and becomes narrower in others. An exact tracing of the system is shown by the solid line and was obtained by a modified Newton’s root algorithm. The shape of the solution in the phase plane depends primarily on the distance of the initial starting values of the system from the origin.

Figure 2 illustrates the behavior of the exact solution of the coupled-regulation model as a function of changes in the three scaling parameters, $a_1$, $a_2$, and $b$. The parameter $k$ is arbitrary. We fixed it at 100 to produce values of the function. As $b$, the instigation or linkage parameter, increases, the first behavior increases relative to the second behavior. On the other hand, increases in the resistance parameter for a particular behavior produce relatively slower increases in that behavior. The combination of these parameters produces the wide variety of forms found in the learned performance literature. In addition to the special cases of the coupled-regulation model, the linear and elliptical forms, the function has a more general skewed form as well, including shapes that seem reminiscent of the rectangular hyperbolic, a form used to fit variable-interval molar response output. Though the flexibility of this function is impressive, some might see it as a liability because it apparently makes the model less vulnerable to disconfirmation. As we point out in the discussion, however, there are a number of ways in which the present model can fail.

Curve Fitting

The present purpose of curve fitting is twofold: first, to provide a methodological approach to fitting such nonstandard functions, and, second, to show that the model is consistent with data from the learned-performance literature. Three points should be emphasized in this exercise: First, other models of learned performance clearly either cannot fit the range of following data (e.g., Allison’s linear model) or, at least, have not attempted to do so. For example, Staddon (1979) fitted only one data set and that one had minimal nonlinearity. Second, curve fitting provides almost no test of nonlinear models unless the data sets are quite large and represent various systematic parametric manipulations. Curves like ellipses and like those of the coupled-regulation models quickly exhaust the degrees of freedom inherent in, for example, six or seven data points.
Figure 2. Three plots showing representative behavior of the coupled regulation model as each of the three parameters of the model change. (For example, in the top left plot are shown the coupled-regulation model's predictions for the relationship between two contingent activities as $a_1$ is increasing. The top right plot shows the same relationships as $a_2$ increases. Finally, the bottom middle plot shows the relationship of the activities as the parameter $b$ increases.)

Finally, because two current models representing extremes in the learned-performance area (Allison, 1976; Staddon, 1979) have both been shown to be special cases of the coupled-regulation model, curve-fitting as a comparative strategy may not be necessary. The coupled-regulation model can fit both linear and elliptical functions. However, to illustrate the differences between the coupled-regulation and the elliptical model we fit one set of data with both. The other data sets show the performance of the coupled-regulation model in response to various schedule and response types.

Methodological Issues

There are several problems in using nonlinear regression in this case. First, the methodology for collecting data from parametric manipulation of schedules of reinforcement is not standardized. Variation in scheduling procedures, species, response measures, and deprivation regimes all provide sources for parameter changes in any model that, at this point, cannot be interpreted easily.

Second, because in most experiments baseline values are not obtained, all models previously discussed gain two free parameters during fitting. The laudable aspect of these models, that of allowing prior information to help predict performance, actually turns out to be disadvantage when that information is unavailable. Staddon's (1979) solution to missing baseline data was to guess at approximate values of the baselines by examining the data and then to fix these values prior to fitting (see, e.g., p. 56, Fig. 6). Although this strategy reduces the number of parameters that must be fit, it does not generally solve the problem and it may introduce parameter biases and reduce the overall fit because it uses the nonoptimal fitting procedure of guessing.

A better approach to reducing the number of parameters that need to be fit when baselines are not specified is to take two passes at the data. The first pass provides estimates
of the parameters that represent the baselines in the model. In the second pass, these estimated baselines are inserted into the equations as constants, that is, they are "frozen" during fitting. Thus, the next pass provides estimates of the three parameters of the model that are independent of variations in the baseline values, although they are dependent on the frozen baseline values. In the following cases when baselines were not provided we used this technique with, typically, not more than a 3% loss of variance accounted for.

Third, as mentioned, the special nature of the coupled-regulation model requires some modification of the usual curve-fitting techniques. Because the form of the model prevents the expression of one activity as a function of the other, a simple regression model cannot be applied. A fitting technique was developed using a modified Newton's root-finding algorithm incorporated into a random-leap curve-fitting program developed by Curry (1975).

**Figure 3** Data (closed circles) collected from rats pressing on a lever working under values of a fixed-ratio schedule. (The dashed line shows the prediction of Staddon's [1979] minimum-distance model, and the solid line shows the prediction of the coupled-regulation model. The square is the predicted baseline determined from Staddon's model and the triangle is the predicted baseline determined from the coupled-regulation model.)

**Fits to Data**

In Figure 3 we compare the fit of the coupled-regulation model and Staddon's elliptical model to the performance of rats on various fixed-ratio values (Kelsey & Allison, 1976). The overall fit of both models is about

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2 The fitting technique requires that the function be represented in a form that allows the parameters to be substituted and then allows the resultant predictions to be compared to the data. The function was written as a homogeneous form, \( f(x) = 0 \), where \( f(x) \) included both sides of Equation 6. During fitting, the parameters and a single data point were both substituted into the homogeneous nonlinear equation, and a Newton's root-finding algorithm was applied to find the predicted value. This value was saved and the procedure was repeated with a new data point having the same parameter values, and the Newton's root-finding algorithm was applied a second time for the second predicted value. This process was repeated for all data points and then a sums of squares was calculated. The fitting program used this sums of squares to either chose new parameters (and began finding a new set of predicted values) to establish a new sums of squares or quit because it had achieved a minimum.
the same, although it is accomplished in different ways. The skew apparent in the data is accommodated by the ellipse by tipping it to the left, whereas the skew is a natural response for the coupled-regulation model. Both models recovered baseline values at very similar points shown at the bottom right-hand side of Figure 3.

Figure 4 shows the performance of a single pigeon across a wide range of ratio values collected by Timberlake and Peden (Note 1). These values were created by manipulating the schedule requirements of both the instrumental and contingent responses. Although the noise level is high, the data do seem to correspond to the model by showing a fast rise, a narrow peak, and a slow decline toward free baseline measures. Fortunately, baseline values were included for these data and could be fixed prior to fitting. Thus, in spite of large data variance, the fit of the coupled-regulation model was statistically reliable.

The next set of data shows a similar form. Figure 5 represents the performance of six chicks (group data) responding for heat at various ratio values (Lucas, Note 2). Note that in this case baseline values were measured in the experiment and fixed prior to fitting.

The final sets of data illustrate the response of the coupled-regulation model to data forms quite different from the previous ones and provide some support for the model as a general account of schedule performance. Figure 6 shows the fit of the coupled regulation model to the key pecking of pigeons for food under a range of variable-interval schedules (Catania & Reynolds, 1968) and fixed-interval schedules (Schneider, 1969). Because neither of these data sets indicated a decline toward some baseline values, the fits were accomplished by arbitrarily picking values for these parameters, and therefore they are not very meaningful. Nonetheless, the form of the coupled regulation model is quite compatible with the data and is reminiscent of the rectangular hyperbolic function that is usually used to fit these data (Herrnstein, 1961, 1970).

There is, however, an important difference between the predictions of the coupled-reg-

![Figure 4](image_url)  
*Figure 4. Data (closed circles) collected from one pigeon working under a variety of fixed-ratio schedules. (Data are from Timberlake & Peden [Note 1]. The solid line shows the prediction of the coupled-regulation model.)*
Figure 5. Data (closed circles) collected from six pigeons working under fixed-ratio schedules for heat reinforcement. (The solid line shows the prediction of the coupled-regulation model. Data were kindly supplied by G. Lucas [Note 2].)

Figure 6. Data (closed circles) collected from six pigeons working under several variable-interval schedules for grain reinforcement (Catania & Reynolds, 1968). (The solid line shows the prediction of the coupled-regulation model, and the dashed continuation shows the prediction of the model extrapolated past the data. Data are also shown (open circles) that were collected from six pigeons working under various fixed-interval schedules for grain reinforcement (Schneider, 1969). The solid line going through these data shows the prediction of the coupled-regulation model, and the dashed continuation shows the prediction of the model extrapolated past the data. VI = variable interval; FI = fixed interval.)
Table 1
Parameter Values for the Coupled-Regulation Model

<table>
<thead>
<tr>
<th>Source</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$b$</th>
<th>$s_1^0$</th>
<th>$s_2^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kelsey and Allison (1976)</td>
<td>$3.6 \times 10^{-1}$</td>
<td>$2.2 \times 10^{-1}$</td>
<td>288.0</td>
<td>8.7</td>
<td>5900.0</td>
</tr>
<tr>
<td>Timberlake and Peden (Note 1)</td>
<td>$1.3 \times 10^{-2}$</td>
<td>0.10</td>
<td>96.0</td>
<td>8.4</td>
<td>84.8</td>
</tr>
<tr>
<td>Lucas (Note 2)</td>
<td>$1.9 \times 10^{-3}$</td>
<td>0.20</td>
<td>0.90</td>
<td>20.0</td>
<td>11.2</td>
</tr>
<tr>
<td>Catania and Reynolds (1968)</td>
<td>1.6</td>
<td>112.3</td>
<td>8.0</td>
<td>1.8</td>
<td>40.0</td>
</tr>
<tr>
<td>Schneider (1969)</td>
<td>32.8</td>
<td>962.0</td>
<td>4.7</td>
<td>.07</td>
<td>40.0</td>
</tr>
</tbody>
</table>

ulation model and the rectangular hyperbolic function. This difference is indicated by the dashed line showing a continuation of the model's predictions, which for some value of $t_2$ abruptly causes $t_1$ to fall. In other words, as reinforcement rate increases, the instrumental activity is predicted to fall abruptly. The values at which this fall occurs in the present data are not meaningful because these data did not include them. However, data at these high rate values for interval and other schedules appear to be critical in testing different models of schedule performance (cf. Baum, 1981).

Finally, Table 1 shows that the parameters of the coupled-regulation model have reasonable values as a function of the variables in the previous experiments. Note first that baseline values were consistent with values obtained in a particular experiment or similar experiments. Generally, the instrumental response was relatively low compared to the contingent response. An exception to these characteristics is the Catania and Reynolds (1968) data, which showed a relatively high rate of baseline pecking. However, it is difficult to say much about this without some data at the high end of the function. The instigation or linkage parameter was generally higher for the ratio schedules relative to the interval schedules. The exception to this pattern was the Lucas (Note 2) data, which showed the smallest instigation. Last, the resistance parameters varied consistently with the baseline values. With the exception of the Kelsey and Allison (1976) data, the contingent response had a greater resistance to change than the instrumental response.

An interesting outcome was the order of magnitude difference between the resistance parameters of the Timberlake and Peden (Note 1) and the Catania and Reynolds (1968) data. The main difference between these experiments was the type of schedule employed. Apparently, the ratio schedule results in a decrease in behavior resistance, whereas the interval schedule seems to promote more behavior resistance. This must stand as speculation because the obtained parameter values could have been due to any number of methodological differences—for example, key resistance, chamber size, and other potential background stimuli. In general, the parameters of the model seem well behaved and may provide a useful summary of schedule variables and their effects.

Discussion

An important contribution of this article was the development of a general mathematical framework for the study of behavior regulation under schedule constraint. This framework stems from the translation of the equilibrium approach advanced by Timberlake (1980) and Timberlake and Allison (1974) into the language of behavior regulation. Central to this approach is the assumption that instrumental and contingent responses form separate regulatory systems that are coupled by a schedule imposed by the experimenter.

Another critical aspect of this framework is that the instrumental and contingent roles assumed by these responses are due primarily to the nature of their schedule-based coupling and not to some intrinsic quality of the responses. It is this symmetric treatment of instrumental and contingent behaviors that separates the present regulatory approach from the majority of other theories of learned performance. Within the equilibrium view,
a schedule changes behavior simply because the response linkage it prescribes violates the response relations shown in free baseline, not because the contingent response has some unique reinforcing quality not shared with other responses. The response linkage imposed by an effective schedule makes it impossible for the organism to realize the base-
lines of both instrumental and contingent response simultaneously. Thus, learned per-
formance represents a compromise between the deviations of the two responses from their respective set points.

A major advantage of this approach is that from a few regulatory assumptions it is possible to predict many results usually attributed to associative-reinforcement processes. Furthermore, in clarifying the role of regul-
atory processes in learned performance, we have provided the context for a more precise designation of the role of associative processes. To this end we suggest that one way that learning may affect responding in our model is by changing the value of $b$, the term that links each response and its coupled-error term. This linkage contributes to responding by amplifying the effects of the coupled-error signal. Thus, instrumental responses that are more closely linked to reward should show greater effect of the disequilibrium imposed by the schedule.

An interesting sidelight of this analysis is that the importance of the coupling repre-
sented by $b$ appears to be asymmetric. The two error signals are modified by reciprocal weightings, $b$ and $1/b$. Thus, if the instru-
mental-to-contingent linkage is high, then the contingent-to-instrumental linkage will be low. Although this assumption is not critical, it stands as a reasonable argument. The transition from contingent-to-instrumental re-
response is typically a punishing one (Timber-
lake, 1980).

A second contribution of this article was the development of a precise and flexible mathematical model of schedule performance. The coupled-regulation model was able to generate functional relations between instrumental and contingent behavior that resemble the variety of forms found in the literature. A closer look at the model's pre-
dictions revealed a reasonable fit to data from simple interval and ratio schedules. No other model we are aware of is able to fit the same variety of forms ranging from linear, ellipti-
cal, and rectangular hyperbolic to the skewed bitonic form of the exact solution of the cou-
pel-regulation model. Also, the model made the apparently unique and testable prediction of a drop in performance on short variable-interval schedules. Finally, from this model we were able to derive as special cases two apparently diverse models of learned performance that are currently cited in the litera-
ture.

A third contribution of this article was the development of an iterative fitting technique that allowed us to estimate the parameters for a homogeneous nonlinear function. This method incorporated a standard equation root-finding technique into a regression algo-

The present method developed for our model is useful when the model can not be expressed in an analytic form [e.g., $y = f(x)$] but can be solved using standard differential equation approaches.

**Commonalities With and Differences From Other Approaches**

Our use of differential equations has a common mathematical basis with the control theories of McFarland and Houston (1980), McDowell and Kessel (1979), and Powers (1978), the behavior-change theory of Atkin-
son and Birch (1970), and the behavior kinetics of Myerson and Miezen (1980). It is not surprising that because we share a com-
mon background in the notion of response deprivation, our approach is most related to the equilibrium-optimality approaches of Staddon (1977, 1979, 1980) and Rachlin and Burkhard (1978). In fact, Staddon (1980) ex-
plicitly used a control-theory model in de-
scribing the determinants of learned performance. He referred to the schedule-feedback function as an output-input function that designated the relation between responding and the effect of that responding in the environment. Similarly, he referred to the control function as an input-output function that described the relation between the consequences of responding and subsequent responding. An important difference from Staddon's analysis is that our model begins with the assumption that there are two separate regulatory systems, one for the instrumental response and one for the contingent response. These two systems are then linked together by the schedule-feedback function to form the final system described by Staddon (1980).

Despite the similarities with previous work, there are several aspects of our approach that merit emphasis. First, we have tried to make explicit the assumptions that went into the development of our differential equations. Most of these assumptions were based on the molar-equilibrium approach to learned performance espoused by Timberlake (1980); however, further assumptions were needed as the development of our equations forced us to clarify the precise nature of the control function. We hope the result of our efforts is a general approach that can be scrutinized at both the level of assumptions and the level of functional form. Alternatives within this approach can be readily developed by altering an assumption or the form of expression of that assumption.

Second, our model represents a clear break with previous work in its lack of dependence on an explicitly derived schedule-feedback function. The role of the schedule-feedback function in coupling the environmental effects of the outputs of the two regulatory systems is clearly vital to our model, but the prior knowledge of the precise form of the linkage is not. As we pointed out, there are advantages to stating the schedule-feedback function only in a general form. Most advantages are related to the problem that the relation between the instrumental and contingent responses specified by the experimenter does not accurately capture the empirical relationship worked out by the animal in interaction with its environment.

On the other hand, there are several potential objections to the absence of a specific schedule-feedback function in our model: (a) Response output cannot be stated as a function of schedule parameters such as programmed rate of reinforcement or ratio size, (b) the moment-to-moment contingencies of responses and reward cannot be specified, and (c) the present work does not continue the long history of research on the local reinforcement determinants of schedule performance.

Oddly enough, these objections can be turned to some advantage. Each objection is based partly on the underlying assumption that learned performance is produced by the response-by-response (or response-interval by response-interval) tracking of probability of reward. In fact, the evidence for this fundamental assumption is mixed. The most successful general approach to fitting learned performance, the matching theory, does not include this assumption in any testable form. Prelec (1982) has recently argued that theories that depend on response-by-response tracking of reward probability may be in difficulty because the local differences between schedules that produce different levels of responding may be too small for the animals to discriminate (cf. Killeen, 1982b).

We think that the framework that we have provided will encourage investigation of the actual form of schedule-feedback functions and of the response or time frame over which they operate. We admit that the absence of a specific schedule-feedback function necessitates the inclusion of parameters that in part serve to estimate its form. However, it is our hope that this general approach can be used to develop a theory of schedule effects that will allow specification of more aspects of the constraint function ahead of time.

Many recent theories of learned performance have a heavy investment in optimality assumptions. A third important aspect of our approach is the lack of dependence on such an optimality analysis. To date, the key characteristics of optimality models have included a dependence on specification of the schedule-feedback function, the exclusive use of stationary analysis to realize the model, and the underlying assumption that all responding is reducible to a single dimension
(Prelec, 1982). Our model shares none of these characteristics. A schedule feedback function was treated only as a device for coupling the outputs of two regulatory systems rather than as a necessary means for apportioning predicted responding between alternatives. Second, we analyzed the differential system dynamically, considering stationary analysis as a special case. Last, we made no assumptions concerning the reducibility of behaviors to some common dimension. As far as the coupled-regulation model is concerned, no relation among responses is needed other than their linkage by the schedule.

The final important aspect of our model is the absence of dependence on an explicit concept of background responding. Although this may seem to be an important omission, it may actually be relatively unimportant. Background responding referred to in other models has not been developed or tested as an empirical concept but was included largely for its response-limiting and scaling properties. Background responding contributes to predictions by competing with the increases in instrumental responding and by allowing rescaling of the importance of instrumental and contingent responding as a function of changes in variables such as deprivation level (e.g., Herrnstein, 1974; Staddon, 1979). In our model these functions are performed by the parameter modifying the resistance of each response to change and by the instigation or linkage parameter that modifies the importance of the error signals. Thus, we may be able to incorporate background responding in our model in a form more consistent with what is actually present.

Areas of Concern

As indicated, an important omission in the present model was the accounting for momentary changes in responding. Part of the basis for this omission was to avoid problems associated with specifying the constraint function. However, there is another potential problem in designating the momentary set points of behavior. The paired baselines serve as measures of total instigation over the entire session; they do not necessarily reflect moment-to-moment changes in instigation. To designate the appropriate momentary set points, it is necessary to discover the time interval over which instigation is integrated to affect behavior and then to estimate the level of instigation over these time periods. Ultimately, studies of the way in which feedback functions contact behavior and studies of the timing and nature of the baseline instigation of responding must be combined to produce a coherent study of local effects.

A second area of concern and an important question in the development of any model of behavior is the role of free parameters. The present model appears to have a large number of such parameters: the two set points \((s^f_0\) and \(s^f_0\)), the index of instigation \((b)\), and the two resistance scales \((a_1\) and \(a_2)\). In most cases, the two set points are fixed by the baseline data prior to imposing the schedule, thus leaving a total of three parameters to be fit from the data. This number of parameters is comparable to the number in most other models that have been proposed to account for learned performance. The number of parameters in a model is not critical as long as the available data contain enough degrees of freedom to estimate these parameters. Further evaluation of the present model and any other model will require a significant number of data points over a large range of schedule values. Such data for interval schedules are notably absent.

A third area of concern is the extension of the model to deal with more complex circumstances involving choice situations and specific background responses. For example, it appears possible to model choice data on concurrent schedules by using a matrix of differential equations that describes the regulatory interplay of four behaviors and their interactions. Although the general choice model has an unwieldy number of parameters, they shrink to a reasonable number given the typical experimental circumstances of the same instrumental and contingent responses for each choice.

Finally, we wish to outline some general ways in which the present model can fail. A model with a flexible form and a number of free parameters may easily assume a life of its own that is reasonably impervious to disproof. Fitting a model to data can become a primary focus of attention, with good fits somehow offsetting bad fits in any overall
evaluation. We tried to make this model susceptible to disproof by clearly stating the assumptions of the model and by developing parameter interpretations based on our intuitions concerning regulatory processes.

The model can be contradicted empirically in many important ways. First, in spite of the flexibility suggested by Figure 2, the form of the model is not compatible with many different forms of data. The model is always bitonic and the rate of rise is constrained by the rate of decline. The location of the peak of the model is a function of the relationship of the set points and the resistance parameters but is independent of the instigation or linkage parameter. The skew apparent in most fits is a natural response of the model; symmetric data (e.g., a normal curve) violate the model.

Second, given a reasonable amount of data from systematic manipulations of schedule parameters, it should be possible to compare precisely the predictions of any similar model with those of our form. For example, although Staddon's (1979) model is related to ours, the two models make different predictions because the center of each function interacts in opposite ways with its peak value. Finally, it is possible to test the model across situations where the parameters are expected to remain invariant. For example, under the uncoupled conditions of baseline assessment, an estimate of the resistance parameters can be obtained that would be expected to be constant during contingency. Not only would this estimate provide a cross-situational test of the model but it would allow the model to fit contingency data with one free parameter.

Summary

We have translated the behavior-equilibrium approach of Timberlake (1980) and Timberlake and Allison (1974) into a theory of behavior regulation based on the concepts of control theory. The fundamental tenet of this approach is that under schedule challenge, learned performance is produced by the simultaneous regulation of behavior with respect to the free-baseline set points of both instrumental and contingent responding. From the behavior-regulation approach, we developed a general mathematical framework for dealing with coupled regulation. We also developed a specific model of two coupled differential equations that described the changes in instrumental and contingent responding under a schedule. Solution of this set of coupled equations produced a complex and flexible function relating instrumental and contingent responding. It was possible to show that two current models of learned performance are special cases of the general coupled-regulation model. Furthermore, the coupled-regulation model provided an adequate fit to a variety of data from simple schedules.

Reference Notes


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